ERC Project PHILOSOPHY OF CANONICAL QUANTUM GRAVITY

Philosophy of Mechanics: Mathematical Foundations

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Abstracts

Mathieu Anel
Introduction to Derived Geometry

Abstract: We will present Derived Geometry as developped by Toën-Vezzosi and Lurie. We will explain the ideas underlying the evolution of the notion of Manifold into Stacks and Derived Stacks (mostly an interpretation of homotopy theory as an ambiguity theory). If time permit we will sketch an application to symplectic geometry.

Gordon Belot
Symmetry, Infinity, and Redundancy

Abstract: A distinction is commonly made between physical symmetries and gauge symmetries: the former can be thought of as relating distinct (but closely related) situations, the latter should be thought of as always relating two descriptions of the same situation, and so as corresponding to a redundancy in the apparatus of the corresponding theory. This distinction is especially clear in the covariant phase space approach to theories like general relativity and classical Yang-Mills-style theories, where it is reflected in the geometric structure of the space of solutions - and where the imposition of asymptotic boundary conditions often implies the existence of physical symmetries and of corresponding conserved quantities. In the case of general relativity, this allows one to make sense of the idea that the physics of isolated systems is time-translation invariant (at least if the cosmological constant is non-positive). The case of Yang-Mills theories is more puzzling.

Daniel Bennequin
Non-abelian (and non-linear) quantization.

Abstract: We generalize the gauge-theoretic interpretation of the quantization process beyond the abelian case by showing that it is possible to quantize a prequantum phase space according to any given Poisson algebra of observables (be it abelian or not, forming a finite dimensional Lie algebra or not). This talk will expose these notions and present some examples. This new framework will permit us to revisit some concepts in symplectic geometry and invariant theory from a shifted point of view.

Jeremy Butterfield
On the Role of Mathematics in the Foundations of Physics.

Abstract: In this talk, I will first discuss Wigner's question about the unreasonable effectiveness of mathematics in the physical sciences. Then I will try to apply this discussion to the role of mathematics in the foundations of physics, especially the foundations of mechanics.

Gabriel Catren
A Portrait of Non-Constrained Mechanics as a Generalized Gauge Theory

Abstract: By using the generalization to non-zero coadjoint orbits of the conjecture by Guillemin and Sternberg according to which ''quantization commutes with (symplectic) reduction'', we argue that quantum phase invariance and gauge invariance have a common geometric underpinning, namely the Mardsen-Weinstein symplectic reduction formalism. From a conceptual viewpoint, this kinship between both kinds of symmetries suggests a gauge-theoretic interpretation of Heisenberg indeterminacy principle. From a formal viewpoint, this gauge-theoretic comprehension of quantum phase symmetries points towards a BRST quantization of ordinary (i.e. non-constrained) mechanical systems. We consider the simplest case of a symplectic manifold endowed with a Hamiltonian action of an abelian Lie group G and we show that the corresponding 0-cocycle equation naturally encodes the polarization of the corresponding quantum states.

John Alexander Cruz Morales
Interplay between model theory, noncommutative geometry and physics after Boris Zilber

Abstract: The main goal of this talk is to review a set of ideas originated in the works of Boris Zilber around the concept of Zariski structure. Following Zilber's approach, after presenting what a Zariski structure is, we will discuss one special type of Zariski structure corresponding to quantum 2-tori and will show how this structure appears in the context of Dirac calculus. This is part of a program which intends to use model theory in order to understand the mathematical content of the structure of the models physicists use for carrying out some of their computations (some times in absence of a rigorous mathematical theory).

Andreas Doering
The real difference between classical and quantum mechanics

Abstract: The self-adjoint operators on a Hilbert space, representing physical quantities of a quantum system, form a noncommutative complex algebra. As is well-known, the noncommutative product of self-adjoint operators itself has no direct physical meaning, but its

anti-symmetrised part (the real Lie algebra given by commutators of self-adjoint operators) and its symmetrised part (the real Jordan algebra given by anticommutators) both have a clear physical interpretation: they relate to time evolution resp. the probabilistic aspects of quantum theory. In classical mechanics, physical quantities are described by measurable, continuous or smooth functions on the state space, which form a commutative algebra. Time evolution also relates to a Lie algebra, given by the Poisson bracket. Yet, the Jordan algebra aspect is trivial in classical mechanics: the Jordan algebra is identical to the commutative algebra of observables; in particular, it is associative. Hence, a key difference between classical and quantum mechanics lies in the real Jordan algebras formed by the physical quantities. Recent work with John Harding shows that the Jordan algebra aspect of a noncommutative von Neumann algebra is fully encoded by the purely order-theoretical structure of contexts, i.e., the partially ordered set of commutative subalgebras of the noncommutative algebra of physical quantities. Moreover, the topos approach to quantum theory provides a natural extension of the state space picture of classical mechanics to quantum theory, with time evolution described by flows on the (generalised) state space.

Frédéric Hélein

The Covariant Phase Space and its Avatars

Abstract: The set of solutions to a problem of the calculus of variations has an universal and canonical property: to be endowed with a pre-symplectic structure. The first geometrization of this property, already detected by Lagrange in his study of mechanics, has been described through the theory of integral invariants of Elie Cartan around 1920, following the work of Poincaré. The fact that this property is also observed in the calculus of variations with several variables has been described for the first time by De Donder in 1935. This discovery, inspired by the theory of Cartan, seems not to have had any significant impact among physicists. However this structure plays a crucial role in the physical field theory. Indeed, it appears as the cornerstone for the construction of the Poisson brackets at work in the quantization of the classical fields. Thus this property was rediscovered several times in various forms : by R. E. Peierls in 1950 by I. Segal in 1960, etc... up to E. Witten, G. J. Zuckerman and C. Crnkovic in 1986. In this talk, I will present this structure in the setting of the multisymplectic formalism, essentially following the most elegant description, according to me, which was given by J. Kijowski and W. Szczyrba in 1976.

Chris Heunen

The many classical faces of quantum structures

Abstract: Much about a quantum system is captured by the collection of its classical subsystems, such as algebraic, logical, and information-theoretic aspects. However, one can rigorously prove that the collection of classical subsystems cannot capture all information about a quantum system, for which more structure has to be added. This question is answered by the notion of an active lattice, which adds the information of how to switch between two classical viewpoints. After a survey of this area, I will discuss how one can reconstruct an operator algebra from its active lattice.

Joseph Kouneiher

Group Cohomology in Physics

Abstract: Topology and geometry have played an important role in our theoretical understanding of quantum field theories. One of the most interesting applications of topology has been the quantization of certain coupling constants. In this lecture, we present a general framework for understanding the quantization of coupling constants in the light of group cohomology. This leads us to choose a group-theoretical approach to quantization that generalizes the geometric quantization formalism. This analysis of some cohomological aspects of physics leads to reconsider the very foundations of mechanics in a new light.

Klaas Landsman

Methods from noncommutative geometry in (constrained) quantization theory

Abstract: Canonical quantum gravity would probably benefit from a number of insights and constructions from noncommutative geometry, which over the last few decades have been applied to abstract constrained quantization theory (but not, so far, specifically to quantum gravity). These include Hilbert C*-modules, Rieffel induction (as the quantum analogue of Marsden-Weinstein reduction in symplectic geometry), and C*-algebras defined by Lie groupoids (with their associated Poisson manifolds defined by Lie algebroids). We give a survey of this development, trying to indicate a possible role in (loop) quantum gravity.

Shahn Majid

Quantum differentials and Poisson geometry

Abstract: I explain the notion of quantum differentials and connections on an algebra and how this can be applied to quantum spacetime in the context of quantum gravity. I also aim to say something about the application of these methods to quantum phase spaces. The main conceptual idea is a new physical degree of freedom that controls the quantum differential structure. A general result will be a functorial first order quantization of geometry over a Poisson manifold. If time, a second result will be the emergence of Riemannian structure out of nothing but the Leibniz rule for quantum differentials. This approach also relates Riemannian geometry to the mathematical structure of BRST quantization.

Charles-Michel Marle

The manifold of motions and total mass of an isolated mechanical system.

Abstract: Around 1810, Lagrange had the idea of considering the set of all possible motions of a mechanical system and studying its properties. He showed that for a conservative system, mathematically described within the framework of classical mechanics, this set has locally the structure of a differential manifold (maybe non-Hausdorff) and is equipped with a natural symplectic form. Jean-Marie Souriau called this set the manifold of motions of the system and clarified Lagrange's results, giving them a global formulation. Mathematical properties—in particular, symmetries—of the manifold of motions of an isolated system (that is to say, free of external influences) obviously depend on the system considered, and also on the mathematical properties assigned to the setting in which the system evolves, namely physical Space and Time. In classical Mechanics, the symmetries of physical Space and Time are described by the Galilean group; this group is thus necessarily contained in the set of symmetries of any isolated mechanical system mathematically described within this theory. The quantities characterizing the system that remain invariant along the motion, such as energy, total momentum or angular momentum, appear as components of a momentum map, with values in the dual of the Lie algebra of the Galilean group. The total mass of the system, which is also conserved along the motion, must be looked for elsewhere: Jean-Marie Souriau showed that it can be interpreted as a symplectic cohomology class for the Galilean group. In Special Relativity, symmetries of Minkowski spacetime are no longer described by the Galilean group, but by the Poincaré group, whose symplectic cohomology is zero. However, a consistent mathematical theory of relativistic Mechanics can still be constructed if one assumes, as did Jean-Marie Souriau, that the manifold of motions of a mechanical system always has a symplectic structure: the total mass of an isolated system then acquires a very different meaning than the one it has in classical mechanics. In both classical and relativistic mechanics, the concept of manifold of motions allows for a sketch of classification of elementary particles, considered as isolated mechanical systems whose manifold of motions is a homogeneous space of the Galilean group (in classical mechanics) or Poincaré group (in relativistic mechanics).

Thierry Masson

Gauge field theories: a comparison of various mathematical approaches.

Abstract: Gauge field theories are the cornerstone of the standard model of elementary particles. Since the discovery of the deep relation between the physical ideas of Yang and Mills and the mathematical structures of connections on principal fiber bundle, various mathematical approaches have been proposed to model gauge field theories. I will make a comparative review of some of these approaches, from ordinary differential geometry, noncommutative geometry, to transitive Lie algebroids.

Julien Page

Towards a Galoisian Interpretation of Heisenberg Indeterminacy Principle

Abstract: We revisit Heisenberg indeterminacy principle in the light of the Galois-Grothendieck theory for the case of finite abelian Galois extensions. In this restricted framework, the Galois-Grothendieck duality between finite K-algebras split by a Galois extension (L:K) and finite Gal(L:K)-sets can be reformulated as a Pontryagin-like duality between two abelian groups. We then define a Galoisian quantum theory in which the Heisenberg indeterminacy principle between conjugate canonical variables can be understood as a form of Galoisian duality: the larger the group of automorphisms of the states in a G-set, the smaller the "conjugate" observable algebra that can be consistently valuated on such states. We then argue that this Galois indeterminacy principle encodes the particular cases of the Heisenberg indeterminacy relations corresponding to the squeezed coherent states that minimize these relations. Finally, we argue that phase group of automorphisms of these squeezed coherent states are given by the groups of the form $H\mathrm{x}H^{\perp}$ (where $H^{\perp}$ is the symplectic orthogonal of the subgroup H of G).

Urs Schreiber

Quantization from Linear homotopy type theory

Abstract: Linear logic has semantics in symmetric monoidal categories, such as that of Hilbert spaces, and hence serves to axiomatize quantum mechanics, e.g. Ambramsky-Duncan (2005), Duncan (2006). We consider its refinement to "linear homotopy type theory" which has semantics in stable infinity-categories of bundles of generalized cohomology theories. We discuss how this provides a natural formalization of (geometric) quantization of local prequantum field theory, following Nuiten (2013). In this context the path integral turns out to be given essentially by the homotopy theoretic and linear refinement of the existential quantifier, the linear dependent sum. Therefore we provide a formal and useful sense in which the statetement "There is a physical trajectory." of classical logic first turns into "The space of all trajectories." in homotopy type theory and then into "The linear space of quantum interfering trajectories." in linear homotopy type theory. This may be thought of as a resolution of the interpretation of quantum superposition and interference in the intuitionistic and linear logic of homotopy type theory.